Diffraction on an acoustically hard disc

Summary of Ir. (M.Sc.) Thesis at Technical University Delft

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Abstract

This Notebook contains an extensive summary of the theoretical and discussion part of my Ir. (Master. Sc.) thesis at the Physics Department of the Technical University Delft (the Netherlands), submitted in June 1963.
It deals with the diffraction and scattering of acoustical waves at a "hard", circular disc in the range $2 < ka < 10$, where $k = 2\pi/\lambda$ is the wave number and a the diameter of the disc. We give the results for the two Kirchhoff approximations and illustrate the theory with experimental data, in particular for the case of oblique incidence of the waves onto the disc. Such data were not available at the time. The experimental results indicate the superiority of the "second Kirchhoff approximation" as an approximate solution for the problem. We also introduce a new quantity, the *random scattering cross section* as a characteristic for the scattering behaviour in the case waves impinge on the scatterer (disc) from all directions. This has a bearing on the effectiveness of scatterers in concert halls and recording studios.
This work was published in the journal Acustica: J.W.M. Baars, "*On the diffraction of sound waves by a circular disc*", 1964, Acustica **14**, 289.
We use *Mathematica* to illustrate the results. In 1963 the patterns had to be numerically calculated using tables of special functions, like Bessel and Struve functions. These are all available in *Mathematica* for easy application.

1. Introduction

Here I summarise the main results of my Ir. (M.Sc.) thesis at the Technical University Delft, the Netherlands, which I completed in June 1963. For this work I was awarded an "encouragement prize" from the Dutch Ministry for Education and Science for the best thesis in physics during 1963. This work was also published in the journal Acustica: J.W.M. Baars, *"On the diffraction of sound waves by a circular disc"*, 1964, Acustica **14**, 289.

The subject is concerned with the scattering of acoustical waves from plane scatterers, as used in concert halls and recording studios. In particular, we wanted to develop a simple quantity to characterise the average scattering cross-section of a scatterer for waves impinging from all directions; a *"random scattering coefficient"*. The work involved both theoretical developments and measurements in an anechoic chamber in order to test the validity of the different theoretical approximations.

An exact solution of the diffraction of a plane wave impinging normally on a circular, acoustically hard disc was obtained by Bouwkamp (1941, 1970) in his Dissertation. He applied spheroidal functions and presented numerical results for several values of $3 < ka <= 10$, where a is the radius of the disc and $k = 2\pi/\lambda$ the wavenumber. Obtaining these solutions is very laborious, especially in the time before software packages like *Mathematica* are available, which provide access to these functions.
I used a few of Bouwkamp's data to see how well my experimental results for normal incidence agree with the exact theory.

Further I studied the two approximate treatments introduced by Kirchhoff, neither of which is rigorously tenable, but they appear to give reasonable results with normal incidence. Considering oblique incidence, the calculations showed rather strong different behaviour between the two approximations. The experimental results then enabled me to demonstrate the superiority of the so-called "second Kirchhoff approximation". I also found a simple expression for the "random scattering cross-section", which was reasonably well represented by the measurements.

In this *Mathematica* Notebook I summarise the mathematical results and use the computing power of *Mathematica* to reproduce some of the theoretical results of my thesis in a much more easy and fast way than was possible in 1963. Colorful graphs illustrate the results; the original figures were handdrawn graphs, black on white.

2. The Kirchhoff diffraction approximations

2.1. Exact - Bouwkamp

2.2. Kirchhoff approximations

We consider the diffraction of an acoustical wave by an acoustically “hard” disc of radius a and area S, located in the x,y-plane (z = 0) with its center in the origin of the coordinate system.. The diffracted wave function satisfies the Helmholtz equation

$$\Delta\Phi + k^2\Phi = 0, \text{ where } k = 2\pi/\lambda, \lambda \text{ is the wavelength.} \quad (1)$$

Using Green’s theorem we obtain as general solution for the diffracted (scattered) wave

$$\Phi_S(P) = \frac{1}{4\pi}\int\int\left\{\Phi_S \frac{\partial}{\partial n}\left(\frac{\exp[ikr]}{r}\right) - \frac{\exp[ikr]}{r}\frac{\partial\Phi_S}{\partial n}\right\} dx\, dy \quad (2)$$

where the integration is extended over the entire x,y-plane containing the disc (z = 0). In the x,y-plane the function Φ_S and its derivative cannot simultaneously be prescribed. As boundary conditions we have:

i) $\Phi_S = 0$ in the plane z = 0 outside of the disc S,

ii) $\frac{\partial}{\partial n}\Phi_S = -\frac{\partial}{\partial n}\Phi_0$ on S.

i) We first assume that the incoming wave impinges **perpendicular** to the disc, traveling along the z-axis.
We seek to reduce the problem to one where the integration is confined to the area S of the disc. This leads to the so-called Kirchhoff approximations.
Making the assumption that $\frac{\partial}{\partial n}\Phi_S = 0$ in the plane z = 0 outside the disc S and using boundary condition ii) on S we obtain the so-called first Kirchhoff approximation

$$\Phi_S^{K1}(P) = \frac{1}{2\pi}\int\int_S \frac{\partial\Phi_0}{\partial n}\frac{\exp[ikr]}{r} dS \;\; , z > 0. \quad (3)$$

Alternatively, we can choose condition i) to be valid and assume that on the "shadow" side of the disc $\Phi_S = -\Phi_0$ ("darkness") to yield the second Kirchhoff approximation

$$\Phi_S^{K2}(P) = -\frac{-1}{2\pi}\int\int_S \frac{\partial}{\partial n}\left(\frac{\exp[ikr]}{r}\right)\Phi_0(x, y, 0)\; dS \;\; , z > 0. (4)$$

In both cases the integration is extended over the area of the disc only.
The variable r is the distance from the field point P to any point on the disc S, whereby P has angular coordinates of θ_P(polar) and φ_P(azimuthal) with respect to the center of S. Assuming P to be sufficiently far from S ("farfield condition") we find the following diffraction functions for the two approximations:

First Kirchhoff $\quad \Phi_S^{K1}(P) = -i\,a\,\frac{J_1(\text{ka}\sin\theta_P)}{\sin\theta_P}, \quad (5)$

Second Kirchhoff $\quad \Phi_S^{K2}(P) = -i\,a\,\frac{J_1(\text{ka}\sin\theta_P)}{\tan\theta_P}. \quad (6)$

ii) Now we reproduce the Kirchhoff functions for **oblique incidence** of the primary wave onto the disc, the angle of incidence with respect to the z-axis being denoted θ_0. The expressions are similar, but the argument now contains the azimuthal angle φ_p of the point of observation P.

First Kirchhoff $\quad \Phi_S^{K1}(P, \theta_0) = -i\,a\,\cos\theta_0\,\frac{J_1(\text{ka}\,\gamma)}{\gamma}, \quad (7)$

Second Kirchhoff $\quad \Phi_S^{K2}(P, \theta_0) = -i\,a\,\cos\theta_P\,\frac{J_1(\text{ka}\,\gamma)}{\gamma}, \quad (8)$

with $\quad \gamma = \sqrt{\left\{(\sin\theta_p\cos\varphi_p - \sin\theta_0)^2 + (\sin\theta_p\sin\varphi_p)^2\right\}}\;. \quad (9)$

Note that for normal incidence ($\theta_0 = 0$) equation 9 simplifies to $\gamma = \sin\theta_P$ and Eqs. (5) and (6) are recovered.

3. The exact diffraction functions and measurements

Before we discuss in some detail the comparisons of the measurements with the two Kirchhoff approximations, we look at how well the experimental setup delivers results in accordance with the exact diffraction function. We present here a few graphs of the diffraction patterns for normal incidence and superpose the measured values. The plots contain bothe the exact curve and the "second Kirchhoff approximation". As we will see later, the second approximation can be considreed the better of the two.
Fig. 1 shows the results for the following valus of ka = 3.2, 4.4, 7.1 and 10.0.

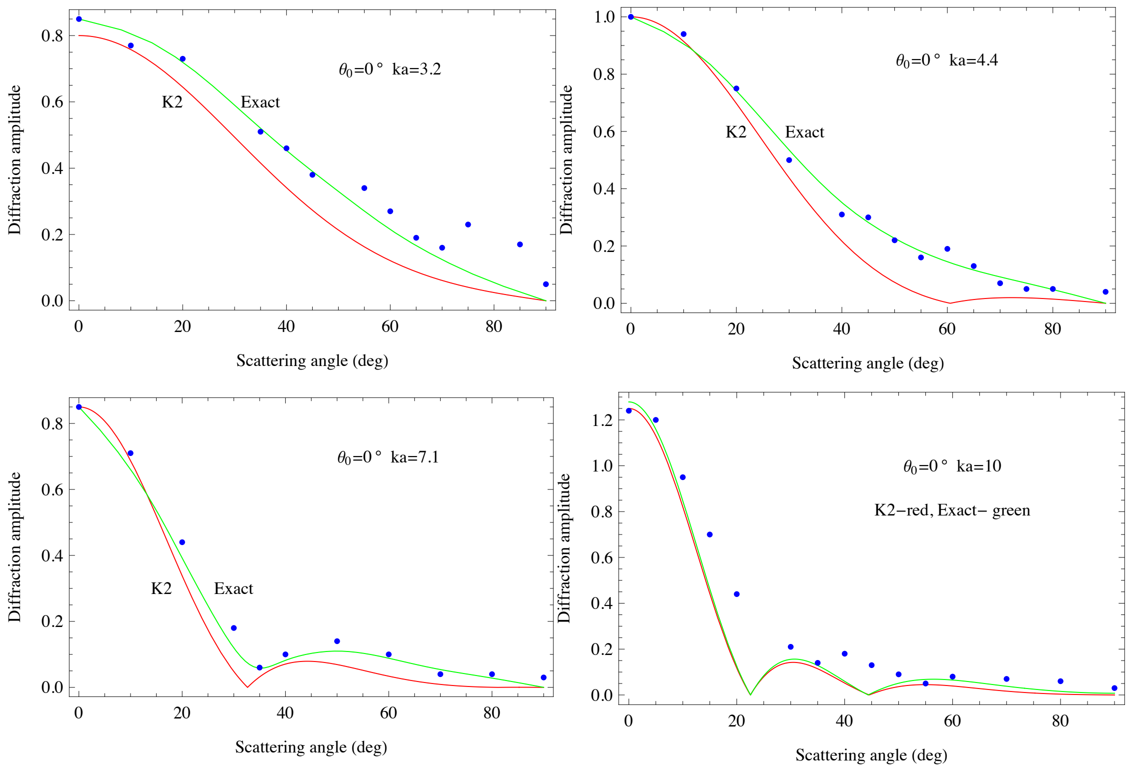
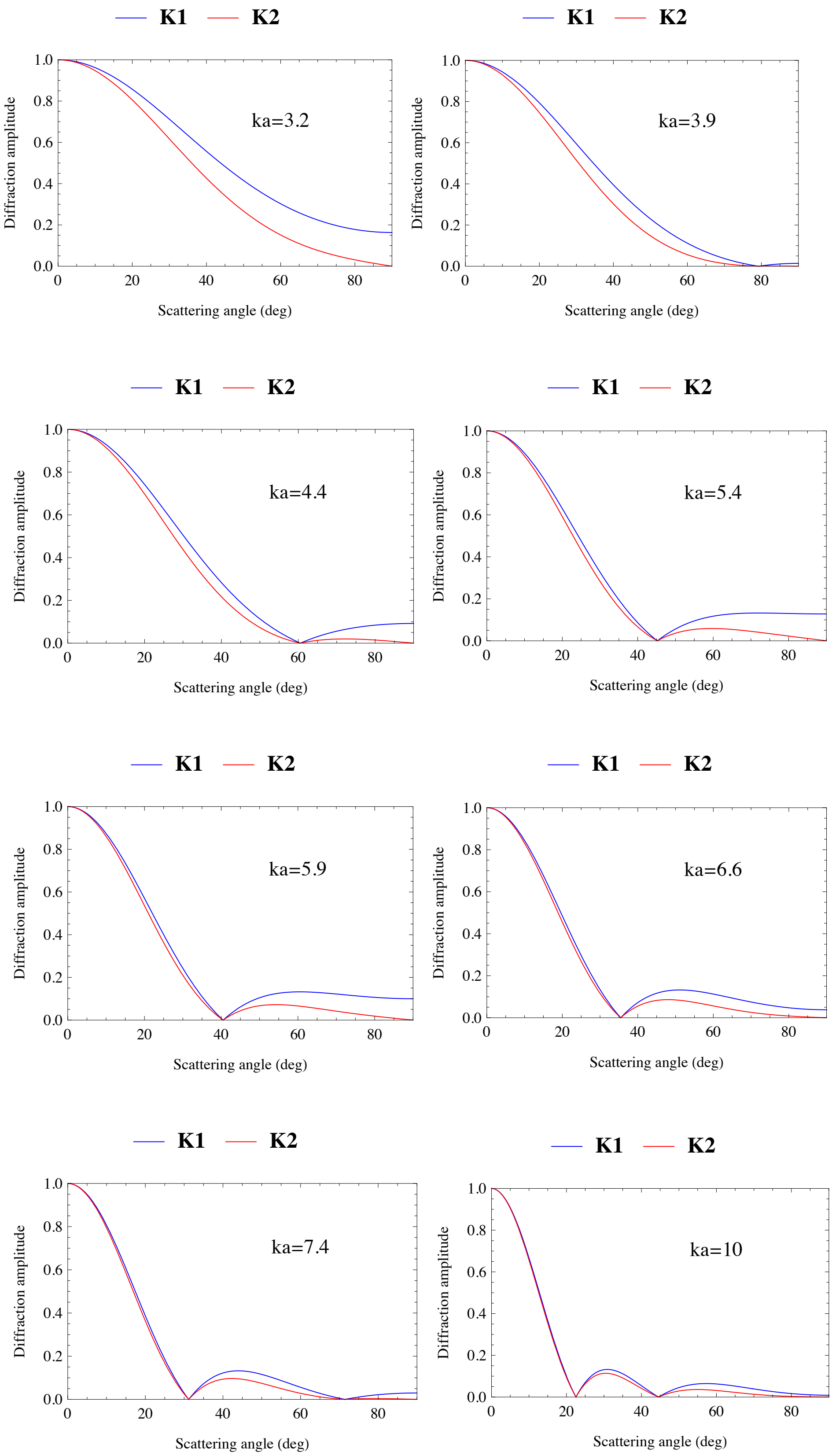


Fig. 1. Diffraction functions for normal incidence on a round disc; exact solution in green, second Kirchhoff appoximation (K2) in red. Blue dots are my measured values. The value of ka is indicated.

First we notice that the K2 solution tends to lie below the exact curve, the difference becoming smaller for larger values of ka. At ka = 10 the difference is very small and it can be siad the the K2 approximation is fully adequate. The measurements of the diffraction amplitude fit the exact theory quite well. The minima for the case ka = 10 hav not been recovered well, perhaps because of an imperfect compensaton of the direct, unscattered field. In any case, we concluded that the measurements were sufficiently reliable to undertake a study of the scattering under oblique incidence.

4. Computations of the Kirchhoff expressions with *Mathematica*

We write now the equations (5) through (9) in the form of a *Mathematica* expression, where we drop the subscript "P" for the angular coordinates of the point of observation. The diffraction functions are denoted K1 and K2 for the first and second Kirchhoff approximation, respectively. The *Mathematica* routines are assembled at the end of this text. In the following three figures the results are presented for normal incidence, 30° and 60° incidence angle, respectively, and for 8 values 3.2 <= ka <= 10.0.
In tose cases under oblique incidence, where we have obtained measurements of the diffraction function, these have been plotted as dots along the calculated curves. This is the case for ka = 3.2, 3.9, 4.4 and 10.0.



For the case of normal incidence we notice that the K1 solution lies somewhat below the K2 curve with the difference becoming smaller for increasing ka. The most remarkable difference is the fact that the K1 solution does not tend to zero for 90° angle, while the K2 approximation does in accordance with the exact theory.Fig. 2. Normalised diffraction function for normal incidence and several values of ka from 3 to 10.

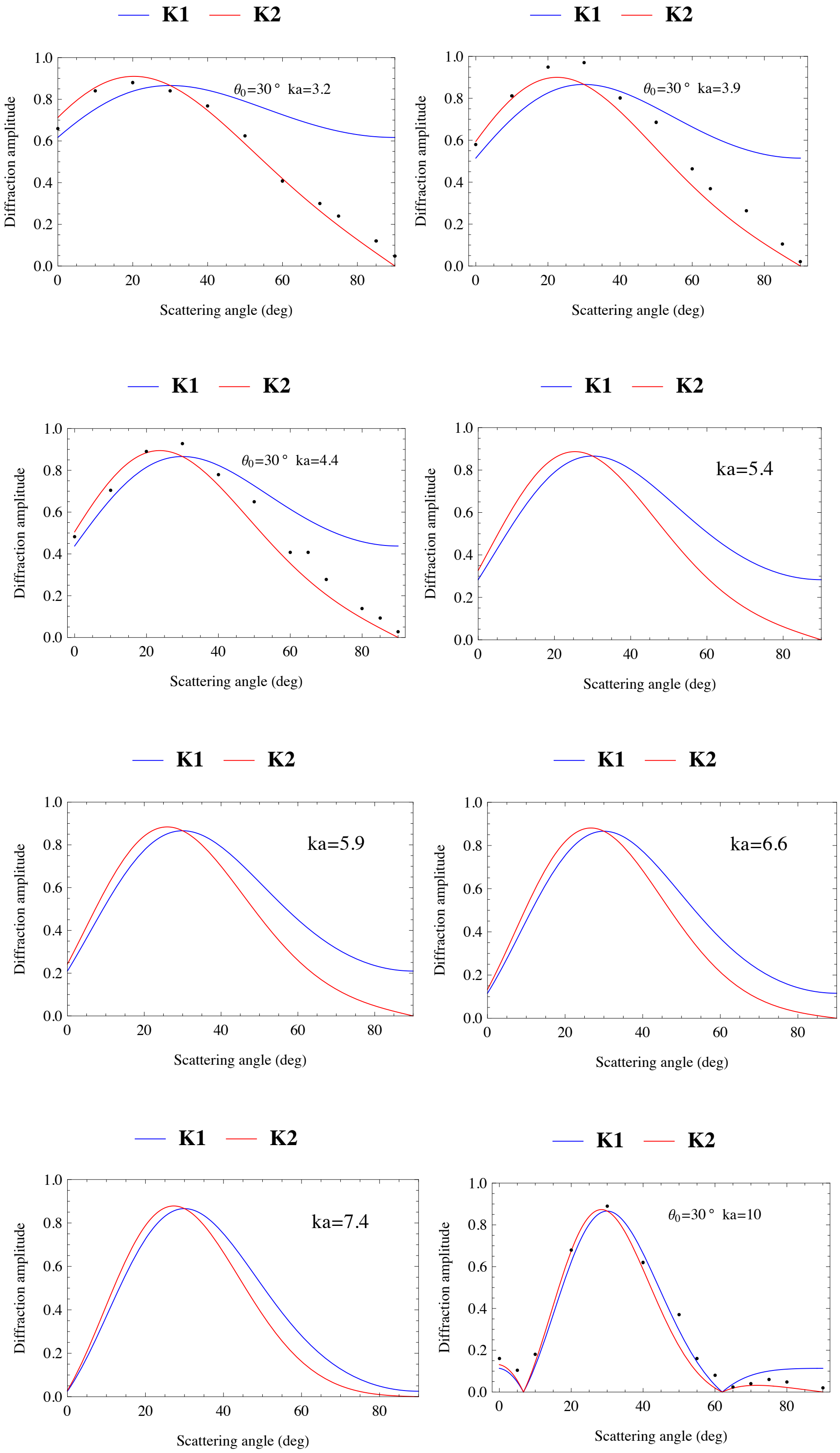


Fig. 3. Diffraction function for 30˚ angle of incidence with measurements (dots), where available.

Fig. 3. shows the Kirchhoff solutions for oblique incidence under an angle of 30°. The difference between the curves is now more pronounced than for normal incidence, in particular for the large angles.Fig. 3. Diffraction function for 30˚ angle of incidence with measurements (dots), where available.

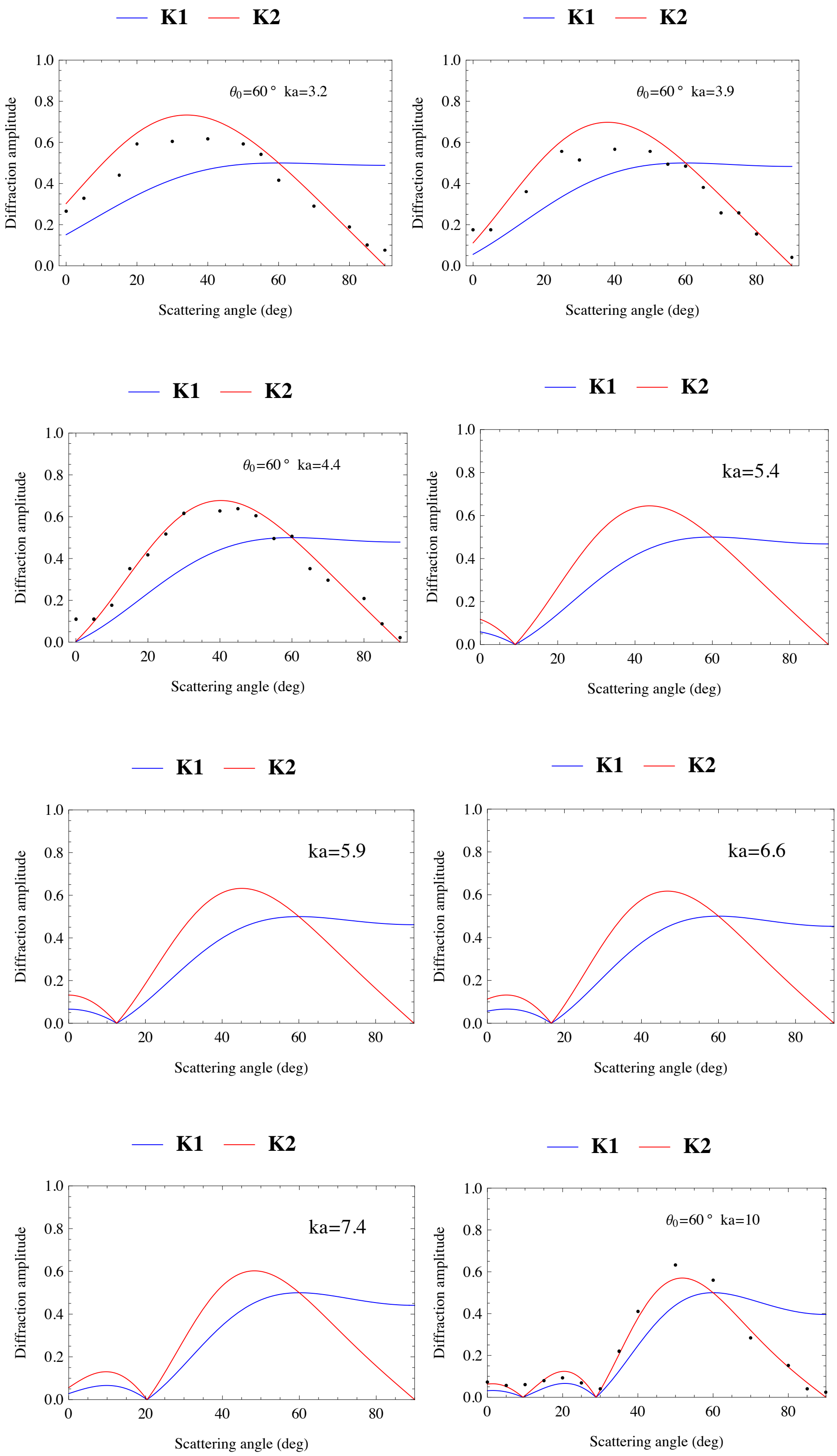


Fig. 4. Diffraction function for 60˚ angle of incidence with measurements (dots), where available.

In the case of oblique incidence under 60˚, as shown in Fig. 4, the two solutions show a completely different behaviour. In particular, the K1 solution remains close to its maximum level from an angle of 50-60 degrees all the way up to 90˚, while the K2 solution descends to zero at 90˚.

5. The experimental results for oblique incidence

As stated earlier, the goal of the measurements was to see in how far the two Kirchhoff approximations represent reality. While the two solutions differ only slightly for normal incidence, the computations indicate significant different behaviour for oblique incidence. Measurements were performed for most of the ka-values used in the computations. The measured diffraction functions have been inserted as dots in figures 3 and 4 for the values ka = 3.2, 3.9, 4.4 and 10.0.
These results clealy show that the second Kirchhoff solution (K2) is a better approximation than K1. The measurements for normal incdence (Fig. 1) indicated this already, be it at a barely significant level. Here the difference between the measurements and the K1 solution is large, in particular for the smaller ka-values. The measured values are generally close to the computed K2 solution. In the case of 30° angle of incidence, the experimental results tend to lie slightly above the K2 curve, but in view of the measurement accuracy the difference is probably not significant. For 60° incidence angle, the situation is reverse for the lower ka-values, where the points lie somewhat below the computed curve. If these differences were real, it is unclear what causes the difference in deviation from positive to negative when going from 30° to 60° angle of incidence.
Our conclusion is that the experimental results under oblique incidence clearly favour the second Kitchhoff approximation for the desription of the diffraction on a circular disc.

6. The scattering cross-section

We now look at the scattering cross-section of the circular disc, which we denote by σ (sigma).
The exact theoretical solution has been presented by Bouwkamp (1940) in the form of spheroidal functions.
He provides a table of values of σ for values of ka = 0 to 10. There is a scarcity of data to represent the function properly, but he nevertheless produces a curve of σ over this range of ka. I have used his tabular data and a spline fitting procedure to construct the plot below.

The general equation for the scattering cross-section S is given by

$$S = -(4\pi/k)\ \mathrm{Im}[A(r_0)], \qquad (10)$$

where $A(r_0)$ is the complex diffraction function in the direction of the primary in-falling wave. In the following we use the normalised scattering coefficient σ, valid for a disc of unit area.
The exact form, as found by Bouwkamp (1940, 1950), is given by

$$\sigma_{\mathrm{ex}} = 2\sum_{n=0}^{\infty} \frac{2}{4n+3} \left|\sigma_{2n+1}\right|^2 , \qquad (11)$$

where $\sigma_n$are the spheroidal coefficients, tabulated by Bouwkamp, and the term in front results from normalisation.

In my thesis I have also calculated the cross-section for the two Kirchhoff approximations. The derivations are rather involved and require significant manipulation with Bessel functions. The resulting expressions are for the first and second Kirchhoff approximation, respectively:

$$S^{\mathrm{K1}} = 2\left(1 - \frac{J_1(2\,\mathrm{ka})}{\mathrm{ka}}\right), \qquad (12)$$

$$S^{\mathrm{K2}} = 2\left(1 + \frac{J_1(2\,\mathrm{ka})}{\mathrm{ka}} - 2J_0(2\,\mathrm{ka}) - \pi\{J_1(2\,\mathrm{ka})\ H_0(2\,\mathrm{ka}) - H_1(2\,\mathrm{ka})\ J_0(2\,\mathrm{ka})\}\right). \qquad (13)$$

Here H_0 and $H_1$are the Struve functions of the zeroth and first order, respectively. They are available in *Mathematica*. In the procedure, the scattering coefficient for a disc with radius one is calculated. We plot the two Kirchhoff approximations of Eqs. 12 and 13 and the exact relation from Bouwkamp S_{ex}as a spline approximation (green curve) to the computed points (red dots) in Figure 5.

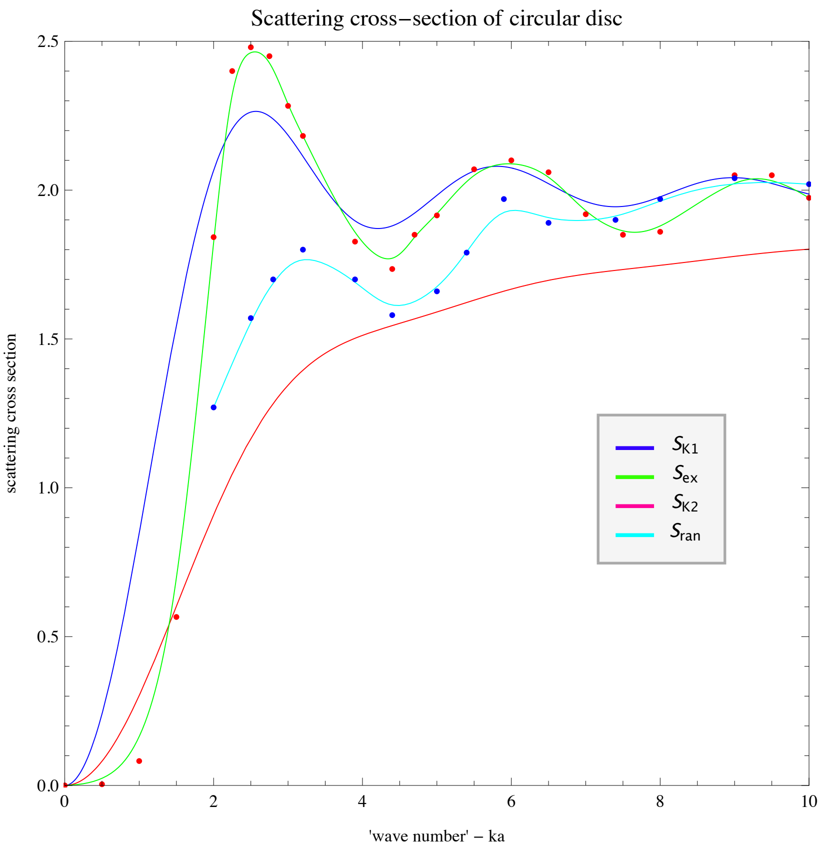


Fig. 5. Scattering cross section of a circular disc of unit radius as function of the parameter ka.
The green curve is a spline approximation to the Bouwkamp exact computations (red dots).
The cyan curve is a spline approximation to the measured “random scattering coefficient”.

It appears that the first Kirchhoff solution (SK1) approximates the exact behaviour quite well, while SK2 is not useful. This is contrary to what we saw in the calculation of the scattering diagrams, where the second Kirchhoff approximation was significantly better.

The reason can be understood by noting that the Kirchhoff method uses different boundary conditions for both cases, which moreover are not exact on part of the integration domain. The calculation of the diffraction function Φ_S involves an integration over the entire x,y-plane. It is reasonable to expect that the best approximation is the one, where the boundary condition is satisfied over as large an area of this plane as possible. This is the case in the second Kirchhoff approximation, where the boundary condition outside the disc is exact $(\Phi_S = 0)$. On the other hand, the total scattered power is determined more directly by the influence of the disc on the impinging wave-fronts. One can now expect the best result for the case where the boundary condition on the disc is exact, as is the case in the first Kirchhoff approximation $\left(\frac{\partial}{\partial n}\Phi_S = -\frac{\partial}{\partial n}\Phi_0\right)$.

Finally, we introduce the scattering coefficient for “randomly” in-falling waves, i.e. waves from all directions in the left hemisphere. Denoting the scattering coefficient for waves impinging under an angle θ as $\sigma(\theta)$, we can define the *“random scattering coefficient”* $\boldsymbol{\sigma}_{\mathbf{ran}}$ as

$$\sigma_{\text{ran}} = 2\int_0^{\pi/2} \sigma(\theta)\,\sin\theta\,d\theta\,. \qquad (14)$$

Unfortunately, we don’t have an analytical expression for $\sigma(\theta)$. We can, however, from our measurements of the diffraction function for oblique incidence determine the field intensity in the direction of incidence. Using these measurements over the angular range 0 to $\pi/2$ radians we can numerically integrate the above equation to find the random scattering coefficient for the ka-values, where data are available.

The plot (in Cyan) with the calculated points from the measurements (blue) is also shown in the above figure. Although numerically smaller than the scattering cross section for perpendicular incidence, the character of the function is similar to that for normal incidence.

7. The *Mathematica* routines and measurement data

```
(* Computation and plot of the two Kitchhoff approximations *)
γ := Sqrt[(Sin[θ °] Cos[ϕ °] - Sin[θ0 °])^2 + (Sin[θ °] Sin[ϕ °])^2];
K1 := Cos[θ0 °] BesselJ[1, ka γ] / γ;
K2 := Cos[θ °] BesselJ[1, ka γ] / γ;
a = 1;
ka =.; θ0 = 0; ϕ = 0;
Table[Plot[Evaluate[{2 Abs[K1] / ka, 2 Abs[K2] / ka},
    {θ, 0, 90}, Frame → True, PlotRange → {{0, 90}, {0, 1.0}},
    PlotStyle → {Blue, Red}, PlotLegends → Placed[{"K1", "K2"}, Above],
    FrameLabel → {"Scattering angle (deg)", "Diffraction amplitude"}],
   Epilog → Inset[Style["ka=" <> ToString[ka], 12], {60, .7}]],
  {ka, {3.2, 3.9, 4.4, 5.4, 5.9, 6.6, 7.4, 10}}]

(* measurements of the diffraction amplitude from 0 to 90 degrees *)

ms3230 = {{0, .66}, {10, .84}, {20, .88}, {30, .84}, {40, .768},
    {50, .624}, {60, .408}, {70, .30}, {75, .24}, {85, .12}, {90, .048}};

ms3260 = {{0, .265}, {5, .328}, {15, .441},
    {20, .592}, {30, .605}, {40, .617}, {50, .592}, {55, .542},
    {60, .416}, {70, .29}, {80, .189}, {85, .101}, {90, .076}};

ms1030 = {{0, .16}, {5, .104}, {10, .18}, {20, .68}, {30, .89},
    {40, .62}, {50, .37}, {55, 0.16}, {60, .08}, {65, .024},
    {70, .04}, {75, .06}, {80, .048}, {90, .02}};

ms1060 = {{0, .072}, {5, .056}, {10, .06}, {15, .08}, {20, .093},
    {25, .068}, {30, .04}, {35, .22}, {40, .41}, {50, .632},
    {60, .56}, {70, .284}, {80, .152}, {85, .04}, {90, .024}};

ms3930 = {{0, 0.58}, {10, 0.812}, {20, 0.949}, {30, 0.97}, {40, 0.8015}, {50, 0.685},
    {60, 0.464}, {65, 0.369}, {75, 0.2636}, {85, 0.1055}, {90, 0.0211}};

ms3960 = {{0, 0.1751`}, {5, 0.1751`}, {15, 0.3605`}, {25, 0.5562`},
    {30, 0.515`}, {40, 0.5665`}, {50, 0.5562`}, {55, 0.4944`}, {60, 0.4841},
    {65, 0.3811`}, {70, 0.2575`}, {75, 0.2575`}, {80, 0.1545`}, {90, 0.0412`}};

ms4430 = {{0, 0.4822}, {10, 0.705}, {20, 0.89},
    {30, 0.927}, {40, 0.7789}, {50, 0.6491}, {60, 0.408}, {65, 0.408},
    {70, 0.278}, {80, 0.1391}, {85, 0.0927}, {90, 0.0278}};

ms4460 = {{0, 0.11}, {5, 0.11}, {10, 0.176}, {15, 0.352}, {20, 0.418}, {25, 0.517`},
    {30, 0.616}, {40, 0.627}, {45, 0.638`}, {50, 0.605}, {55, 0.495`},
    {60, 0.506`}, {65, 0.352}, {70, 0.297}, {80, 0.209}, {85, 0.088}, {90, 0.022}};

(* plot of the Kirchoff functions with
 experimental results to be merged here from above
  adjust the ka and θ parameters and the plot names in the routine below! *)
ka = 3.2; θ0 = 60; ϕ = 0;
γ := Sqrt[(Sin[θ °] Cos[ϕ °] - Sin[θ0 °])^2 + (Sin[θ °] Sin[ϕ °])^2];
K1 := Cos[θ0 °] BesselJ[1, ka γ] / γ;
K2 := Cos[θ °] BesselJ[1, ka γ] / γ;
p360 = Plot[Evaluate[{Abs[K1], Abs[K2]} 2 / ka, {θ, 0, 90}],
   Frame → True, PlotRange → {{0, 90}, {0, 1.0}},
   PlotStyle → {Blue, Red}, PlotLegends → Placed[{"K1", "K2"}, Above],
   FrameLabel → {"Scattering angle (deg)", "Diffraction amplitude"}, Epilog →
    {Prepend[Point /@ meas, PointSize[0.01]], Inset["θ0=60°  ka=3.2", {60, 0.85}]}]
```

```
(* scattering cross sections; exact (Bouwkamp),
Kirchhoff (two) and the "random scattering cross section as measured *)
a = 1;
S1 := 2 (1 - BesselJ[1, 2 k a] / (k a));
S2 := 2 (1 + BesselJ[1, 2 k a] / (k a) - 2 BesselJ[0, 2 k a]
      - π (BesselJ[1, 2 k a] StruveH[0, 2 k a] - StruveH[1, 2 k a] BesselJ[0, 2 k a]));
g1 = Plot[{S1, S2}, {k, 0, 10}, Frame → True, AspectRatio → 1,
    PlotRange → {{0, 10}, {0, 2.5}}, PlotStyle → {Blue, Red},
    PlotLabel → "Scattering cross-section of circular disc",
    FrameLabel → {"'wave number' - ka", "scattering cross section"},
    Epilog -> Inset[Panel[Grid[MapIndexed[
          {Graphics[{Hue[(1 - 4 First@#2) / 10], Thick, Line[{{0, 0}, {1, 0}}]},
             AspectRatio → .1, ImageSize → 20], #1} &,
          {"SK1", "Sex", "SK2", "Sran"}]]], {8, 1}]];
(*  data from Bouwkamp *)
data = {{0, 0}, {.5, .004}, {1, .082}, {1.5, .566}, {2, 1.842}, {2.25, 2.4},
   {2.5, 2.48}, {2.75, 2.45}, {3.0, 2.283}, {3.2, 2.182}, {3.9, 1.827},
   {4.4, 1.735}, {4.7, 1.85}, {5, 1.915}, {5.5, 2.07}, {6.0, 2.1}, {6.5, 2.06},
   {7.0, 1.919}, {7.5, 1.85}, {8.0, 1.86}, {9.0, 2.05}, {9.5, 2.05}, {10, 1.974}};
g2 = Graphics[{Green, BSplineCurve[data], Red, Point[data]}];
(* measured random cross sections against ka*)
mcs = {{2., 1.27}, {2.5, 1.57}, {2.8, 1.7}, {3.2, 1.8},
    {3.9, 1.7}, {4.4, 1.58}, {5., 1.66}, {5.4, 1.79}, {5.9, 1.97},
    {6.5, 1.89}, {7.4, 1.9}, {8., 1.97}, {9., 2.04}, {10., 2.02}};
ran = Graphics[{Cyan, BSplineCurve[mcs], Blue, Point[mcs]}];
Show[g1, g2, ran]
```

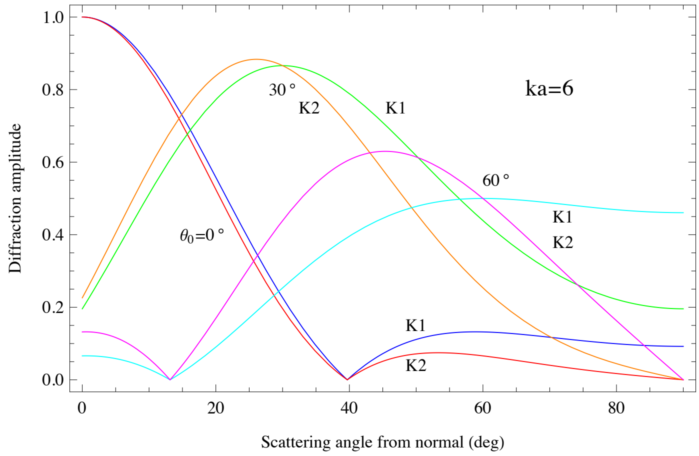


Fig. 5. Composite plot for both Kirchhoff solutions, ka = 6 and angles of incidence 0˚, 30˚ and 60˚.

Appendix - some of Bouwkamp's numerical data in *Mathematica*

exact and K2 comparison

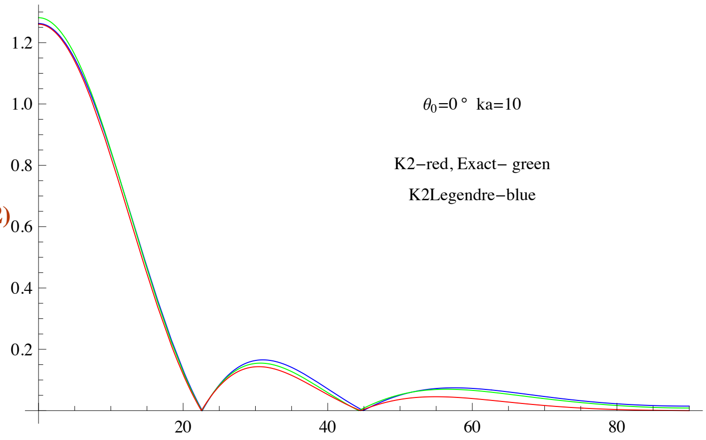
```
ex44 = {{0, 100}, {10, 93}, {20, 75}, {30, 53},
    {40, 34}, {50, 22}, {60, 14}, {70, 9}, {80, 5}, {90, 0}};
m44 = {{0, 100}, {10, 94}, {20, 75}, {30, 50}, {40, 31}, {45, 30}, {50, 22},
    {55, 16}, {60, 19}, {65, 13}, {70, 7}, {75, 5}, {80, 5}, {90, 4}};
mc44 = Table[{m44[[i, 1]], m44[[i, 2]] / 100}, {i, 1, 14}];
exc44 = Table[{ex44[[i, 1]], ex44[[i, 2]] / 100}, {i, 1, 10}];
```

```
ex32 = {{0, 85}, {10, 82}, {20, 73}, {30, 59},
   {40, 45}, {50, 33}, {60, 21}, {70, 12}, {80, 5}, {90, 0}};
m32 = {{0, 85}, {10, 77}, {20, 73}, {35, 51}, {40, 46}, {45, 38},
   {55, 34}, {60, 27}, {65, 19}, {70, 16}, {75, 23}, {85, 17}, {90, 5}};
mc32 = Table[{m32[[i, 1]], m32[[i, 2]] / 100}, {i, 1, 13}];
exc32 = Table[{ex32[[i, 1]], ex32[[i, 2]] / 100}, {i, 1, 10}];

exm71 = {{0, 85, 85}, {10, 69, 71}, {20, 39, 44}, {30, 10, 18}, {35, 4, 6},
   {40, 9, 10}, {50, 12, 14}, {60, 9, 10}, {70, 5, 4}, {80, 3, 4}, {90, 0, 3}};
ex71 = Table[{exm71[[i, 1]], exm71[[i, 2]] / 100}, {i, 1, 11}];
m71 = Table[{exm71[[i, 1]], exm71[[i, 3]] / 100}, {i, 1, 11}];

exm10 = {{0, 124, 124}, {5, 115, 120}, {10, 75, 95}, {15, 45, 70},
   {20, 15, 44}, {30, 17, 21}, {35, 4, 14}, {40, 8, 18}, {45, 3, 13},
   {50, 6, 9}, {55, 10, 5}, {60, 9, 8}, {70, 5, 7}, {80, 3, 6}, {90, 0, 3}};
ex10 = Table[{exm10[[i, 1]], exm10[[i, 2]] / 100}, {i, 1, 15}];
m10 = Table[{exm10[[i, 1]], exm10[[i, 3]] / 100}, {i, 1, 15}];

ka = 10; θ0 = 0; ϕ = 0;
γ := Sqrt[(Sin[θ °] Cos[ϕ °] - Sin[θ0 °]) ^2 + (Sin[θ °] Sin[ϕ °]) ^2];
K1 := Cos[θ0 °] BesselJ[1, ka γ] / γ;
K2 := Cos[θ °] BesselJ[1, ka γ] / γ;
p10 = Plot[Evaluate[1. * Abs[K2] / ka, {θ, 0, 90}],
   PlotRange → All, PlotStyle → Red, Frame → True,
   FrameLabel → {"Scattering angle (deg)", "Diffraction amplitude"}, Epilog →
    {Inset["θ₀=0°  ka=10", {60, 1.0}], Inset["K2-red, Exact- green", {60, .8}]}];
splex10 = Graphics[{Green, BSplineCurve[ex10], Blue, Point[m10]}];
Show[p10, splex10]
```

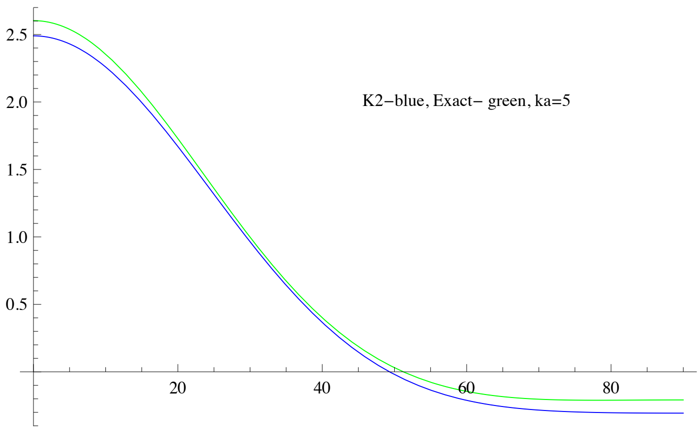
Exacte en Kirchhoff buiging aan een ronde schijf - Bouwkamp dissertatie

```
α2n = {{0, .18, .1746}, {1, .34, .3828}, {2, .96, .934}, {3, 1.80, 1.753},
   {4, 1.24, 1.266}, {5, .43, .481}, {6, .09, .112}, {7, .01, .0224}};
ka = 10; θ0 = 0; ϕ = 0;
γ := Sqrt[(Sin[θ °] Cos[ϕ °] - Sin[θ0 °]) ^2 + (Sin[θ °] Sin[ϕ °]) ^2];
K1 := Cos[θ0 °] BesselJ[1, ka γ] / γ;
K2 := Cos[θ °] BesselJ[1, ka γ] / γ;
pK2 = Plot[Evaluate[2.52 Abs[K2] / ka,
    {θ, 0, 90}, PlotStyle → Red, PlotRange → All, Frame → True,
    FrameLabel → {"Scattering angle (deg)", "Diffraction amplitude"}]];
pK2B = Plot[0.25 * Abs[Sum[α2n[[i, 2]] * LegendreP[2 (i - 1), Cos[ θ π / 180]],
      {i, 1, 8}]], {θ, 0, 90}, PlotRange → All, PlotStyle → Blue];
pexe = Plot[Abs[0.25 * Sum[α2n[[i, 3]] * LegendreP[2 (i - 1), Cos[ θ π / 180]],
      {i, 1, 8}]], {θ, 0, 90}, PlotRange → All, PlotStyle → Green];
Show[pK2B, pexe, pK2, Epilog → {Inset["θ₀=0°  ka=10", {60, 1.0}],
   Inset["K2-red, Exact- green", {60, .8}], Inset["K2Legendre-blue", {60, .7}]}]
```

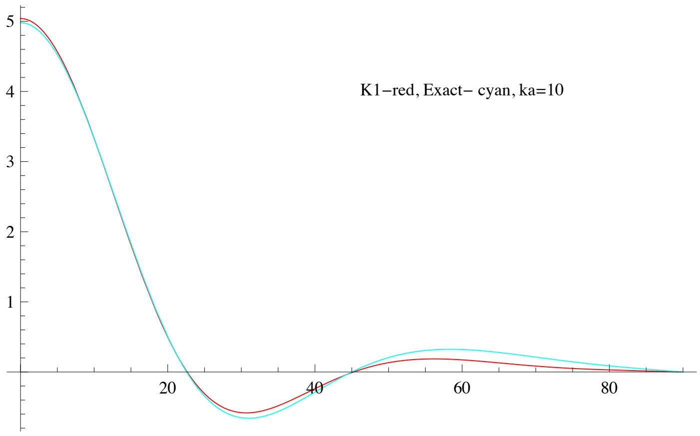
(27.12.12)




```
ka = 5;
α2n = {{0, .14, .208}, {1, 1.39, 1.355},
   {2, .79, .8338}, {3, .16, .1836}, {4, .01, .0224}};
pK2 = Plot[Sum[α2n[[i, 2]] * LegendreP[2 (i - 1), Cos[ θ π / 180]], {i, 1, 5}],
   {θ, 0, 90}, PlotRange → All, PlotStyle → Blue];
pex = Plot[Sum[α2n[[i, 3]] * LegendreP[2 (i - 1), Cos[ θ π / 180]], {i, 1, 5}],
   {θ, 0, 90}, PlotRange → All, PlotStyle → Green];
Show[pK2, pex, Epilog → Inset["K2-blue, Exact- green, ka=5", {60, 2}]]
```




```
αn = {{0, .32, .37}, {1, .6, .42}, {2, 1.4, 1.54},
   {3, 1.56, 1.63}, {4, .84, .76}, {5, .26, .22}, {6, .06, .04}};
pK1 = Plot[Sum[αn[[i, 2]] * LegendreP[2 ( i - 1) + 1, Cos[ θ π / 180]], {i, 1, 7}],
   {θ, 0, 90}, PlotRange → All, PlotStyle → Red];
pexo = Plot[Sum[αn[[i, 3]] * LegendreP[2 (i - 1) + 1, Cos[ θ π / 180]], {i, 1, 7}],
   {θ, 0, 90}, PlotRange → All, PlotStyle → Cyan];
Show[pK1, pexo, Epilog → Inset["K1-red, Exact- cyan, ka=10", {60, 4}]]
```




```
ex32 = {{0, 85}, {10, 82}, {20, 73}, {30, 59},
   {40, 45}, {50, 33}, {60, 21}, {70, 12}, {80, 5}, {90, 0}};
m32 = {{0, 85}, {10, 77}, {20, 73}, {35, 51}, {40, 46}, {45, 38},
   {55, 34}, {60, 27}, {65, 19}, {70, 16}, {75, 23}, {85, 17}, {90, 5}};
mc32 = Table[{m32[[i, 1]], m32[[i, 2]] / 100}, {i, 1, 13}];
exc32 = Table[{ex32[[i, 1]], ex32[[i, 2]] / 100}, {i, 1, 10}];
```